Homomorphisms of graphs to odd cycles

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RESEARCH MEMORANDUM

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HOMOMORPHISMS OF GRAPHS TO ODD CYCLES

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HOMOMORPHISMS OF GRAPHS TO ODD CYCLES

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ABSTRACT. We give a class of graphs G for which there exists a homomorphic (= adjacency preserving) map from V(G) to V(C), where C is the shortest odd cycle in G. Hereby extending a result of Albertson, Catlin and Gibbons. Moreover our class of graphs is characterized by the property: For each odd subdivision of G' there exists a homomorphic map from V(G') to V(C') where C' is the shortest odd cycle of G'.

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1. INTRODUCTION

A graph G' is called a homomorphic image of a graph G if there exists a map $\varphi : V(G) \to V(G')$ preserving adjancency, i.e. if $uv \in E(G)$ then $\varphi(u)\varphi(v) \in E(G')$ ($V(G)$ denotes the nodeset of G, $E(G)$ its edgeset). We say G *maps to* G'. In this paper we are interested in maps to odd cycles. Obviously, if G maps to an odd cycle of length M then no odd cycle in G is shorter than M. The result of this paper is

<u>THEOREM</u>. *Let* G *be a non-bipartite graph. Then either* G *maps to its shortest odd cycle or* G *contains an odd-*K_4 *or an odd-*K_3^2 *subgraph.* □

Here an *odd*-K_4 and an *odd*-K_3^2 are graphs as indicated in fig. 1. Wriggled and dotted lines stand for (pairwise openly disjoint) paths, dotted lines may have length zero and *odd* indicates that the corresponding faces are odd cycles.

fig. 1

We shall give a proof in section 2. As pointed out in the remark at the end of that section the proof yields a polynomial-time algorithm which finds a map to the shortest odd cycle of G or finds an odd-K_4 or an odd-K_3^2.
The theorem implies the following result due to Albertson, Catlin and Gibbons |1984|. Here *fold* means repeated identification of nodes at distance two.

COROLLARY 1. *Let* G *be a graph, and let* M *be an odd number* ($\geqq$ 3) *such that* G *contains no odd cycle shorter than* M. *Then either* G *maps to a cycle of length* M *or* G *contains a subgraph which folds to an odd-*K_4 *in which all odd cycles have length* M.

PROOF (that corollary 1 follows from the theorem).
If G is bipartite it maps to each odd cycle. So assume G is not bipartite. Let K be the length of its shortest odd cycle. Assume G does not map to a cycle of length M. Since each odd cycle of length greater than or equal to M maps to a cycle of length M, G does not map to a cycle of length K. By the theorem G has a subgraph G' such that G is an odd-K_4 or an odd-K_3^2. It is not hard to see that G' folds to a odd-K_4 in which each odd cycle has length M. □

Note that the theorem properly extends corollary 1 since the graph in fig. 2 folds to K_4 (the 4-clique), but it does not contain an odd-K_4 or an odd-K_3^2.

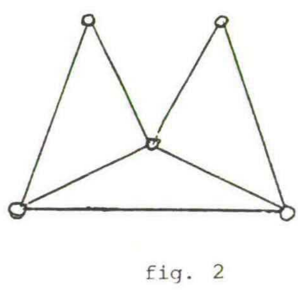

fig. 2

As another corollary of the theorem we get a characterization of the graphs not containing an odd-K_4 or an odd-K_3^2. The proof is left to the reader as and easy exercise. In [Gerards, Schrijver, 1985b] one can find more characterizations of these graphs. Using one of these characterizations Schrijver obtained a short and elegant proof of the main theorem of this paper. However that characterization relies on Tutte's characterization of regular matroids (Tutte [1958]), whereas the proof given in section 2 is elementary.

Odd-subdivision means: replacing edges by paths of odd length.

COROLLARY 2. *A graph* G *does not contain an odd-*K_4 *or an odd-*K_3^2 *iff* G *is bipartite or each odd-subdivision* G' *of* G *maps to the shortest odd cycle of* G'. □

REMARK. From the theorem it follows: If G contains no odd-K_4 and no odd-K_3^2 then the length of a shortest odd cycle equals the maximal cardinality of a packing of E(G) with sets of the form $\langle X\rangle \cup \langle V(G)\backslash X\rangle$, $X \subset V(G)$. Here $\langle X\rangle$ denotes the set of edges with both ends in X. A weighted version of this min-max relations follows via odd-subdivision if all weights are odd.
In fact Seymour [1977] showed that this weighted form holds for any weight function and for any signed graph not reducible to the signed graph in fig. 3. For signed graphs and reductions of them see the first part of section 2.

2. PROOF OF THE THEOREM

Before we get to the actual proof we give a preliminary result. It is convenient for its proof to state it in terms of signed graphs.
A *signed graph* (G,E_0) is an undirected graph G together with a subset, E_0, of the edgeset E(G). The edges in E_0 are called *odd*, the other edges are called *even*. A cycle C in G is called *odd* (*even*) if $E_0 \cap E(C)$ is *odd* (*even* respectively). A signed graph is *bipartite* if it contains no odd cycles. Obviously odd-K_4 and odd-K_3^2 can be defined in this context as well. (Then the faces in fig. 1 indicated with *odd* are signed odd cycles.)
The following proposition is easy to check.

PROPOSITION. *A signed graph* (G,E_0) *contains an odd-*K_4 *or an odd-*K_3^2 *iff* G *can be reduced to* K_4 *(with all edges odd) or to the graph in fig. 3 (bold edges even, thin edges odd).* □

Reduction means: deletion of edges, contraction of even edges, or *resigning*, i.e. replacing E_0 by $E_0 \Delta B$, where B is a minimal cut (Δ := "symmetric difference").

fig. 3

REMARK. Not that also holds "(G,E_0) contains an odd-K_4 iff (G,E_0) can be reduced to K_4". But "(G,E_0) contains an odd-K_3^2 iff (G,E_0) can be reduced to the signed graph in fig. 3" is not true.

We call a cycle C in a (signed) graph *non-separating* if for each two edges e and f not in E(C) there exists nodes $v_1,\dots,v_k$ not on C such that v_1 is contained in e, v_k is contained in f, and v_j and v_{j+1} are adjacent ($j = 1,\dots,k-1$). So C is separating if removing C from G (including the nodes of C) topologically disconnects G.

LEMMA. *Let* (G,E_0) *be a signed graph not containing an odd-*K_4 *or an odd-*K_3^2. *Let* C *be a non-separating odd cycle in* (G,E_0). *If the edges induced by the nodes not on* C *form a bipartite signed graph than all odd cycles in* G *contain one fixed node of* G.

PROOF.

If $V(G)\backslash V(C) = \emptyset$, then each edge in $E(G)\backslash E(C)$ is a chord of C. Since C is non-separating, $|E(G)\backslash E(C)| \leqq 1$ in which case the lemma follows trivially. So let us assume: $V(G)\backslash V(C) \neq \emptyset$. Let T be a tree in G with $V(T) = V(G)\backslash V(C)$ (T exists since C is non-separating). Delete the edges induced by $V(G)\backslash V(C)$ not on T. Resign such that $E(T) \cap E_0 = \emptyset$. Contract the edges in E(T). Now

each odd cycle in G contains an odd cycle in the contracted graph, which by the proposion does not contain an odd-K_4 or an odd-K_3^2. Hence we may assume G to be the contracted graph. So $V(G) = V(C) \cup \{w\}$ (w not in $V(C)$).
Let C' be an odd cycle in G, such that $|E(C') \cap E(C)|$ is minimal. Choose $u \in V(C) \cap V(C')$. We show that each odd cycle in G contains u. Suppose to the contrary that odd cycle C" does not contain u. By the minimality of $|E(C') \cap E(C)|$ we have that $E(C') \cap E(C) \cap E(C") = \emptyset$. There are five possibilities now, indicated in fig. 4. In any of them G contains an odd-K_4 or an odd-K_3^2.

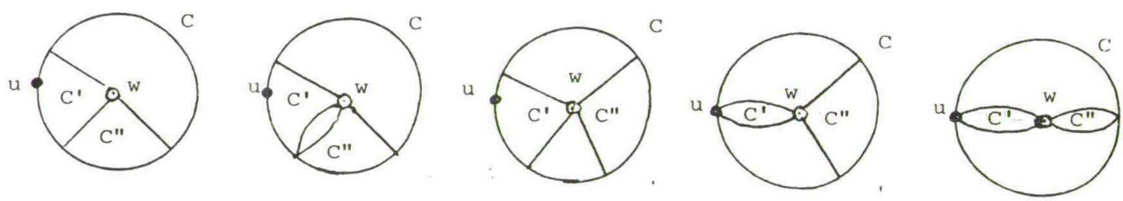

fig. 4

□

Now we leave the context of signed graphs. If we refer to the previous part of this section, we do this by refering to "the proposition" or "the lemma", and we assume $E_0 = E(G)$.

<u>PROOF OF THE THEOREM</u>. Let G be a minimal counterexample. Let M be the lenght of its shortest odd cycle. $\mathcal{S}$ denotes the collection of shortest odd cycles in G. It is not hard to see that in a minimal counterexample, as G is, there cannot be a separating shortest odd cycle. Let $E_{\mathcal{S}}$ denote the set of edges contained in a shortest odd cycle. Let $G_1 = (V_1,E_1)$, $G_2 = (V_2,E_2),\ldots, G_k = (V_k,E_k)$ be the components of $(V(G),E_{\mathcal{S}})$. (Note that (V_i,E_i) may be equal to $(\{v\},\emptyset)$ for a $v \in V(G)$.)

Claim 1. *Let* $i = 1,\dots,k$. *If* Γ_1 *and* Γ_2 *are odd cycles in* G *such that* $V(\Gamma_1) \cap V_i \neq \emptyset$ *and* $V(\Gamma_2) \cap V_i \neq \emptyset$ *then* $V(\Gamma_1) \cap V(\Gamma_2) \neq \emptyset$,

Proof of claim 1. Assume Γ_1 and Γ_2 are odd cycles, $V(\Gamma_1) \cap V_i \neq \emptyset$, $V(\Gamma_2 \cap V_i) \neq \emptyset$ and $V(\Gamma_1) \cap V(\Gamma_2) = \emptyset$.
Since G_i is connected there exist $C_1, C_2, \dots, C_\ell$ in G_1, $C_j \in \mathcal{S}$ $(j = 1,\dots,\ell)$ such that $V(\Gamma_1) \cap V(C_1) \neq \emptyset$, $V(C_j) \cap V(C_{j+1}) \neq \emptyset$ $(j = 1,\dots,\ell-1)$, and $V(\Gamma_2) \cap V(C_\ell) \neq \emptyset$. Without loss of generality we may assume: $\ell=1$. Let F be the set of edges in $E(\Gamma_1) \cup E(\Gamma_2)$ which do not meet C_1. Then F is a forest. Extend F to a tree T such that $V(T) = V(G)\setminus V(C_1)$ (that is possible as C_1 is non-separating). Then the lemma applies to C_1 as a subgraph of $E(T) \cup E(\Gamma_1) \cup E(\Gamma_2) \cup E(C_1)$. Hence $V(\Gamma_1) \cap V(\Gamma_2) = \emptyset$, contradicting our assumption.

end of proof of claim 1

We are now going to prove that k=1, i.e. $(V,E_{\mathcal{S}})$ is connected. We do this by contradiction.

Assumption: $k \geqq 2$.
Let B be the set of edges leaving V_1. Since G is connected, $B \neq \emptyset$. Moreover $e \in B$ implies that there is no $C \in \mathcal{S}$ with $e \in E(C)$.

Claim 2. *If* Γ *is an odd cycle in* G, *then* $|E(\Gamma)\setminus B| \geqq M$.

Proof of claim 2. Let Γ be a counter example with $|E(\Gamma) \cap B|$ minimal and, under that restriction, $|E(\Gamma)\setminus B|$ minimal. Obviously $|E(\Gamma) \cap B|$ is not 0 or 2. Hence $|E(\Gamma) \cap B| \geqq 4$.
First we prove

(*) *If* $C \in \mathcal{S}$, $u,v \in V(C) \cap V(\Gamma)$ *with* $u \neq v$. *Then* $\min\{|E(\Pi_1) \cap B|, |E(\Pi_2) \cap B|\} = 0$, *where* Π_1 *and* Π_2 *are the paths on* Γ *with endpoints* u *and* v.

Assume (*) is not correct. Let P_1 and P_2 be the paths on C with endpoints u and v, such that $|E(P_1)| \equiv |E(\Pi_2)| \pmod 2$. Then for $i = 1,2$, $E(P_i) \,\Delta\, E(\Pi_i)$ contains an odd cycle Γ_i, say, such that $|E(\Gamma_i) \cap B| \leqq |E(\Pi_i) \cap B| < |E(\Gamma) \cap B|$. Hence $|E(P_i)| + |E(\Pi_i)\setminus B| \geqq |E(\Gamma_i)\setminus B| \geqq M$ (by the minimality of Γ). Since $|E(P_1)| + |E(P_2)| = M$ we now have $|E(\Gamma)\setminus B| = |E(\Pi_1)\setminus B| + |E(\Pi_2)\setminus B| \geqq M$, contradicting our assumption that Γ violates claim 2, hence (*) is correct.

Now let $v_1, v_2 \in V_1 \cap V(\Gamma)$ such that the two paths Π_1 and Π_2 on Γ with endpoints v_1 and v_2 satisfy $E(\Pi_1) \cap E_i = \emptyset$, $|E(\Pi_1) \cap B| = 2$. Let C_1 and C_2 be shortest odd cycles in G_1 with $v_1 \in V(C_1)$ and $v_2 \in V(C_2)$, and such that $|E(C_i) \cap E(\Gamma)|$ is maximal for $i = 1,2$. Then $|V(C_i) \cap V(\Gamma)| = 1$ or $E(C_i) \cap E(\Gamma)$ is a path.

Indeed, let $u \in V(C_i) \cap V(\Gamma)\backslash\{v_i\}$. Let Q be the path on Π_2 from v_i to u, and let P be the path on C_i from v_i to u, such that $|E(P)| = |E(Q)|$.

Since $C_1 \in \mathcal{S}$: $|E(Q)| \geqq |E(P)|$. Let Γ' be an odd cycle in $(E(\Gamma)\backslash E(Q)) \cup E(P)$.

By (✱) $E(Q) \cap B = \emptyset$; hence $|E(\Gamma') \cap B| \leqq |E(\Gamma) \cap B|$ and $|E(\Gamma')\backslash B| \leqq |E(\Gamma)\backslash B| - |E(Q)| + |E(P)|$.

By the minimality of Γ we now have $|E(Q)| \leqq |E(P)|$, so $|E(Q)| = |E(P)|$.

Hence $(E(C_i)\backslash E(P)) \cup E(Q)$ is a shortest odd cycle, which violates the maximality of $|E(C_i) \cap E(\Gamma)|$ unless P=Q.

Now let $b \in E(\Pi_2) \cap E(B)$ (which is not empty). Let Π_{21} and Π_{22} be the components of $E(\Pi_2)\backslash\{b\}$ with $v_i \in V(\Pi_{2i})$, $i = 1,2$. By (✱) $V(\Pi_{21}) \cap V(C_2) = \emptyset$ and $V(\Pi_{22}) \cap V(C_1) = \emptyset$. Hence $V(\Gamma) \cap V(C_1) \cap V(C_2) = \emptyset$.

Since $V(C_1) \cap V(C_2) \neq \emptyset$ (claim 1) and since $E(C_i) \cap E(\Gamma)$ is a path, the cycle Γ is non-separating in the union of Γ, C_1 and C_2. So by the lemma there exists an odd cycle C, say, in $E(C_1) \cup E(C_2)$ such that $V(C) \cap V(\Gamma) = \emptyset$.

From the fact that C_1 and C_2 are in $\mathcal{S}$ it is not hard to see that $C \in \mathcal{S}$, and $(E(C_1) \cup E(C_2))\backslash E(C)$ is also a cycle in $\mathcal{S}$. This cycle violates (✱).

<u>end of proof of claim 2</u>

Now we contract the edges in B. We get a graph G' which does not contain an odd-K_4 or an odd-K_3^2 (by the proposition), and in which each odd cycle has length greater than or equal to M (claim 2). By the minimality of G, G' maps to a cycle of length M. Equivalently there exists a map $\varphi' : V(G') \to \mathbb{Z}$ such that $|\varphi'(u) - \varphi'(v)| \equiv 1 \pmod M$ for each $uv \in E(G')$.

Define $\varphi : V(G) \to \mathbb{Z}$ by $\varphi(u) = \varphi'(u')$ if $u \notin V_1$ and $\varphi(u) = \varphi'(u') + 1$ if $u \in V_1$, where u' is the node in G' to which u is contracted.

<u>Claim 3.</u> $|\varphi(u) - \varphi(v)| \equiv 1 \pmod M$ *for each* $uv \in E(G)$.

<u>Proof of claim 3.</u> Let $u,v \in V(G)$, $uv \in E(G)$. If $uv \in B$ then u and v are contracted into the same node of G', and $u \in V_1$, $v \notin V_1$ (or vice versa). Hence $\varphi(u) = \varphi(v) + 1$. If $uv \notin B$ then u and v are both in V_1 or both not in V_1. Moreover they are not contracted to one node since then there would be

an odd cycle Γ in G with $|E(\Gamma)\backslash B| = 1$, violating claim 2. So $|\varphi(u) - \varphi(v)| \equiv |\varphi'(u') - \varphi'(v')| \equiv 1 \pmod M$.

<u>end of proof of claim 3</u>

Hence G maps on a cycle of length M, contradicting our assumption that G violates the theorem, so our assumption $k \geqq 2$ is not correct. Take any odd cycle $C \in \mathcal{S}$. By claim 1 it meets any odd cycle in G. Since C is non-separating there exists a $v \in V(C)$, such that each odd cycle in G passes through v (see the lemma). Hence the graph induced by $V\backslash\{v\}$ is bipartite. Let $V_r \cup V_b$ be a bipartition. Denote the set of nodes of $V\backslash\{v\}$ which have distance 0 to v by V^i. The collection $\{V^i | i = 1,\ldots,\lfloor \frac{1}{2}M \rfloor\}$ is a partition of $V\backslash\{v\}$ as each $u \in V$ is in an odd cycle of length M. Moreover $V^i \cap V_r$ and $V^i \cap V_b$ are stable sets. Contracting each of them to a single node yields a cycle of length M (this contraction is a homomorphism!). So again G maps to its shortest odd cycle, thus providing a final contradiction. □

<u>REMARK</u>. It is not hard to deduce from the proof a polynominal-time algorithm which determines, given a graph G with no odd-K_4 or odd-K_3^2, a homomorphic map from G to its shortest odd cycle. Indeed, the length of a shortest odd cycle, as well as a shortest odd cycle containing some fixed node or edge can be determined in polynomial-time. Hence so can $E_{\mathcal{S}}$ (Grötschel, Pulleyblank [1981] or Gerards, Schrijver [1985a]). Moreover, if we find a map to a cycle of length M after we contracted the edges leaving some component of $(V,E_{\mathcal{S}})$, we can easily find such a map for the original graph. In fact we can use the above proof to create a polynomial-time algorithm which determines, given a graph G, a homomorphic map from G to its shortest odd cycle, an odd-K_4, or an odd-K_3^2. That algorithm starts with the algorithm indicated above. If that fails (e.g. claim 2 does not hold, or $E_{\mathcal{S}}$ is connected but there is no $v \in V(G)$ such that $V(G)\backslash\{v\}$ induces a bipartite graph) then it is possible to reconstruct the proof in polynomial-time, such that one finds an odd-K_4, an odd-K_3^2 or a separating shortest odd cycle. In the last case a decomposition of the problem is possible. Since the details are rather complicated, and messy, we shall omit them.

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REFERENCES

[1984] M.O. Albertson, P.A. Catlin and L. Gibbons, *Homomorphisms of 3-chromatic graphs*, II, preprint.

[1985a] A.M.H. Gerards and A. Schrijver, *Matrices with the Edmonds-Johnson property*, Report No. 85363, Inst. für Ökon. u. Op. Res., Uni. Bonn, W. Germany.

[1985b] A.M.H. Gerards and A. Schrijver, in preparation.

[1981] M. Grötschel and W.R. Pulleyblank, *Weakly bipartite graphs and the max-cut problem*, Oper. Res. Letters 1 (1981), 23-27.

[1977] P.D. Seymour, *The matroids with the max-flow min-cut property*, J. Combinatorial theory (B) 23 (1977) 189-222.

[1958] W.T. Tutte, *A homotopy theorem for matroids*, II, Trans. Amer. Math. Soc., 88 (1958) 161-174.

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